Research Statement

Aleš Janka

My past and present research activities consist in proposing and studying efficient numerical methods applicable to resolution of a wide range of partial differential equations (PDEs) and optimization problems. In particular, in the past four years, I have been developing

- *algebraic multigrid methods by smoothed aggregation*: an efficient multigrid solver of sparse systems of linear equations. The main application fields have been compressible and incompressible fluid simulations, linear elasticity, and electromagnetism.
- *shape optimization algorithms*: conception of multi-level parametrization techniques to accelerate convergence of standard optimization methods. The main application area was the aerodynamic design of a small business jet for supersonic and transonic régimes.
- *mesh generation algorithms*: a non-Delaunay approach for automatic anisotropic mesh generation, applied then to produce a set of coarser meshes for setup of geometric multigrid.

Algebraic multigrid by aggregation

Speeding up resolution of sparse linear systems of equations presents a rewarding challenge, as most of the CPU time in engineering simulation and optimization processes is spent for inverting sparse matrices. One of the few known optimal iterative solvers is the multigrid method, exploiting the physical nature of the underlying problem; by neglecting fine resolution scales one constructs a coarse (and thus easier to solve) problem which serves to speedup the convergence. At the same time, however, this very connection to the nature of the physical problem (and to its discretisation) causes most of the multigrid algorithms to be case-specific, tightly connected to the simulation code, requiring huge amount of data for setup like, for example, the sequence of coarser meshes, and direct access to discretization techniques and implementation of boundary conditions on coarse levels.

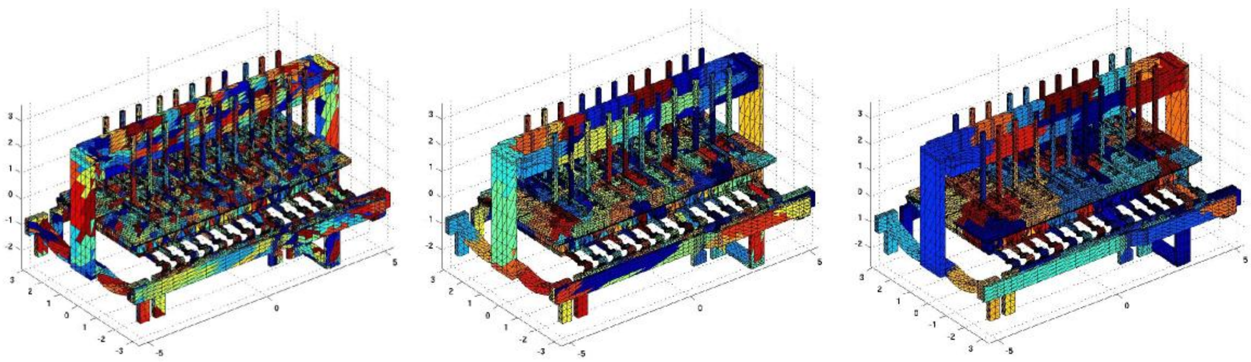

Figure 1: Algebraic multigrid: representation of nodal aggregates which define three successive coarse levels for solution of electric potential in an aluminium electrolysis cell

My past and present research in this domain

During my PhD thesis and a post-doctoral stay at the Institut National de la Recherche en Informatique et Automatisation (INRIA Sophia Antipolis, France) we proposed and analysed a better multigrid scheme with a setup which is much cheaper and more versatile than standard multigrid methods - the algebraic multigrid by aggregation.

The new method is completely decoupled from the underlying discretization of the physical problem; all information needed is obtained only from the system matrix itself, from a set of local kernel functions (e.g. the rigid-body motion modes for linear elasticity) and the type of problem solved. Especially, there is no mesh information involved; coarsening is done by agglomerating the set of degrees of freedom using the neighbourhood information from the sparsity pattern of the system matrix, see Fig. 1.

- Within my PhD thesis at INRIA Sophia Antipolis, I collaborated with Dr. Petr Vaněk (Math Dept, University of California, Los Angeles) and Dr. Hervé Guillard (INRIA) in order to analyse the convergence of the proposed solver. We have extended the convergence results of the abstract theoretical framework of Bramble [5], which uses a Ritz-Galerkin technique to formulate coarse problems, to a Petrov-Galerkin approach [1] more appropriate for treating compressible fluid flow problems with boundary layers.
- During my post-doctoral appointment at Ecole Polytechnique Fédérale de Lausanne, I proposed an extension of this method for indefinite problems [2], encountered e.g. in simulations of incompressible flows, constrained optimization or mixed finite element methods. Also, we have designed a new domain-decomposition method for solving problems on unbounded domains, which exploits the proposed multigrid scheme. The main field of application are simulations of ferromagnetic effects [3] and calculation of magnetic induction. The above methods have been or are being integrated into an engineering expert code dedicated to simulation of aluminium electrolysis (Alcan-Péchiney company).

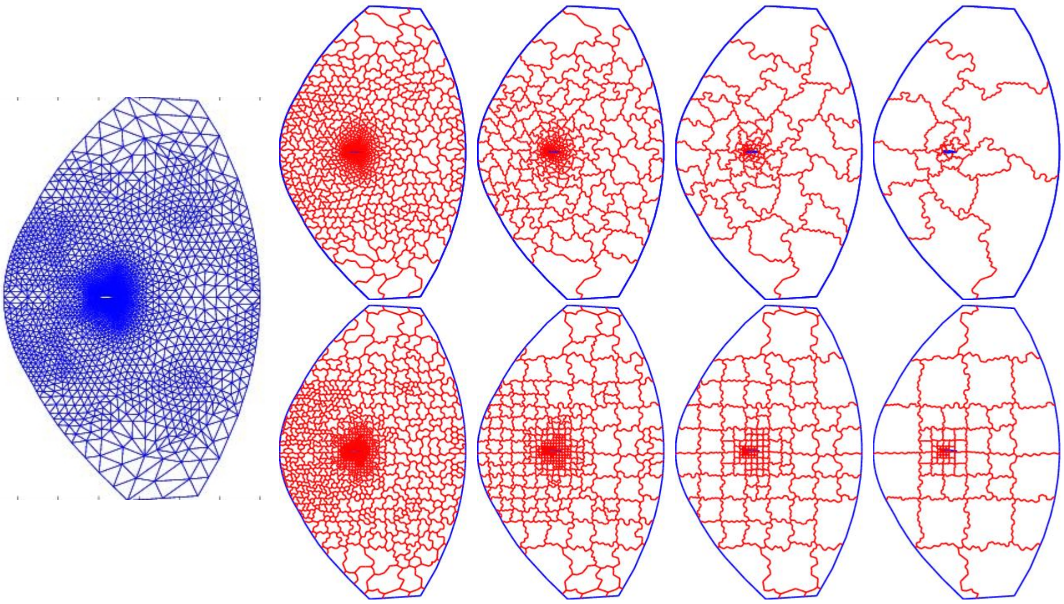

Figure 2: Unstructured finite element mesh of a wing profile (left), representation of hierarchy of multigrid levels by nodal aggregation: arbitrary nodal aggregates on levels 2 to 5 (top line), vs improved nodal aggregates on levels 2 to 5 (bottom line) producing regular stencil matrices for coarse levels with better CPU-cache access.

Perspectives for future work

- The efficiency of the proposed scheme has been demonstrated, both theoretically and experimentally, for symmetric problems. Open questions remain, however, for non-symmetric problems which cannot be considered as perturbations of symmetric ones (e.g. for flows with very high Reynolds numbers). In this case, classical reasoning in terms of error-dampening known from multigrids for elliptic problems does not apply. One shall investigate the phase error [8], instead of dampening, to account for propagation of error out of the computational domain.

- As to computer implementation, although multigrid approach gives theoretically an optimal method, their low CPU-cache efficiency when used for industrial engineering problems with unstructured meshes is an obstacle to successful scaling for very large problems. The cache efficiency depends significantly on the used sparse matrix storage scheme. While the sparsity pattern of the matrix is given by the discretization of the PDEs on unstructured meshes and could be unfavourable to parallel cache operations, the multigrid by aggregation could be designed to produce (structured) matrices of coarse problems which are much favourable to cache access optimization. Recently, a Hierarchical Hybrid Grid cache-aware method was presented [6], which scales well for up to $1.7 \cdot 10^{10}$ unknowns. It is, however, based on geometric multigrid, which is in my opinion too connected with mesh and physics. Combined with aggregation of unknowns, we could develop a method which scales well and is quite versatile at the same time.

Shape optimization techniques

Considering the parallel between the minimization of a shape cost-function and iterative solution of PDEs (which might also be considered as a minimization of energy or finding a saddle-point of a functional), one concludes that multigrid techniques might as well apply in the context of shape optimization.

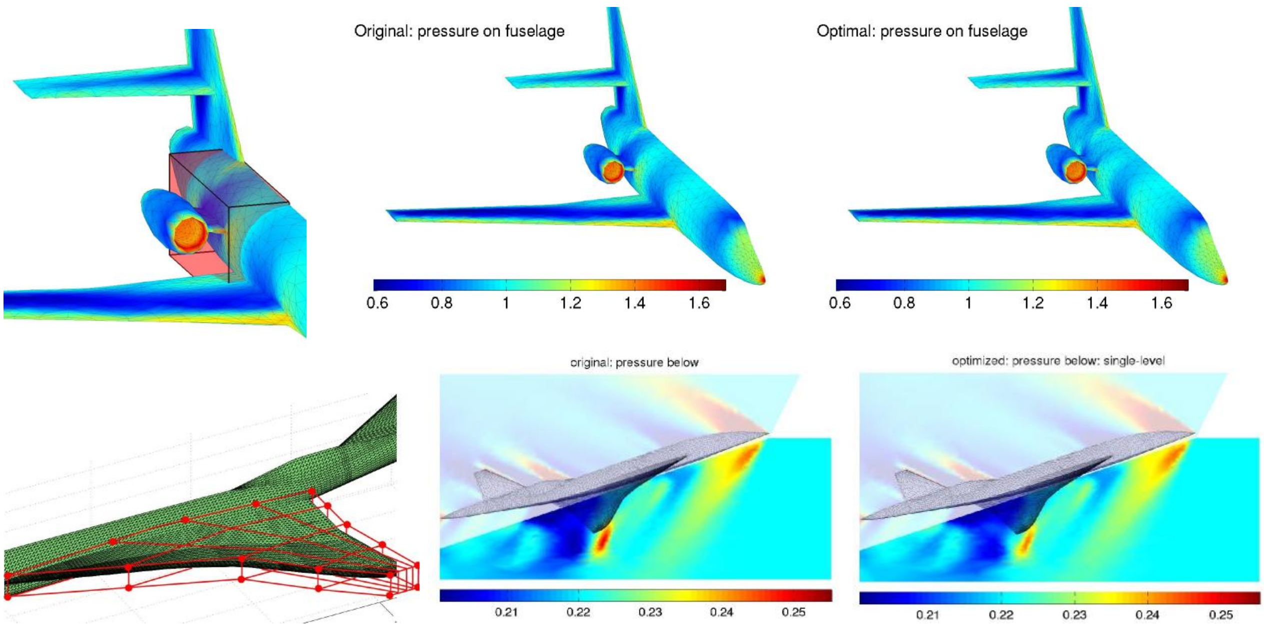

Figure 3: Shape optimization: parametrization by Free-Form Deformation, original and optimized pressure fields for a transonic engine-pylon-fuselage pressure smoothing (top) and supersonic business jet pressure-shock reduction below the wing

My past and present research in this domain

During my one year post-doctoral appointment at INRIA Sophia Antipolis with Dr. Jean-Antoine Désidéri I worked on the project "Réseau de recherche et d'innovation technologique – Recherche Aéronautique sur le Supersonique", whose aim was to minimize the noise below a small supersonic business jet. We continued to apply the same technique in the collaboration with Piaggio Aero Industries (Dr. Lanari, Ing. Dario Pinelli from Finale-Ligure, Italy) for improving the design of a small transonic business jet Piaggio P180. Our approach is based on the following ingredients:

- Use of a Free-Form Deformation (FFD) approach (originating from Computer Graphics) as a very versatile parametrization technique for complex 3D forms in shape optimization. While in the conventional techniques one parametrizes the surface, in this technique we parametrize deformations of a volume around the optimized form.
- A sequence of embedded Bézier tensorial parametrizations of the FFD volume, from the finest to the coarsest, served as a basis for a multi-level optimization, in which standard simple optimization techniques were used like pre- and post-smoothers in a multigrid method. Our method was shown faster and more robust (convergence to a "better" minimum) than the common mono-level approach [4].
- Together with Dr. Abderrahmane Habbal (Université de Nice) we experimented with hybridization of fast and precise descent methods efficient for finding local minima even for a large number of design parameters, with a costly but robust evolutionary algorithms able to advance towards the global minimum, applicable to reduced number of parameters.

Perspectives for future work

- Due to high computational costs, even the best shape optimization codes try to work with the least number of design parameters. Usually, the pertinent parameters are identified by an a-priori engineering knowledge of the designer. During my post-doctoral stay we also started to experiment with an automated re-parametrization of the optimized shape for simple model problems. In this way, we achieved better solution with the same computational effort.
- Recently, the industrial optimization community is becoming more and more interested in a "robust design", i.e. design which is performing well for an interval of uncertainty for problem parameters (e.g. the Reynolds and Mach numbers for shape optimisation in aerodynamics), as opposed to optimality only for fixed parameter values. This brings up new questions of sensitivity with respect to these parameters, together with the need of even more efficient optimization algorithms. Several options are to be explored:
  - Penalization of the sensitivity to problem parameters [7],
  - Game theory for players with two different interests - shape minimization and insensitivity to parameter change,
  - Min-max approach: shape optimisation for worst possible values of problem parameters.

Mesh generation algorithms

During a short period of 9 months after my PhD I worked still under the direction of Dr. Hervé Guillard on a Dassault Aviation project of automatic anisotropic mesh coarsening (in 2D and 3D) used for creating a sequence of node-nested coarser meshes for a geometric multigrid. Due to additional constraints on the remeshing process (like preserving the structured nature of boundary layer, mesh anisotropy of up to aspect ratio 10^5, insertion of no new nodes, and a given local anisotropic coarsening ratio) the task is, in my opinion, more difficult than a standard mesh adaptation.

My past and present research in this domain

In collaboration with the Centre de Mise en Forme CEMEF, Ecole des Mines de Paris, we generalized for our purposes their existing non-Delaunay topology-preserving meshing tool MTC. In the MTC mesher, the quality of each simplex element (triangle or tetrahedron) is iteratively optimized on a space of local topological changes. The quality of an element is measured by its correspondence to an equilateral simplex, viewed in a metric field given on nodes of the original mesh.

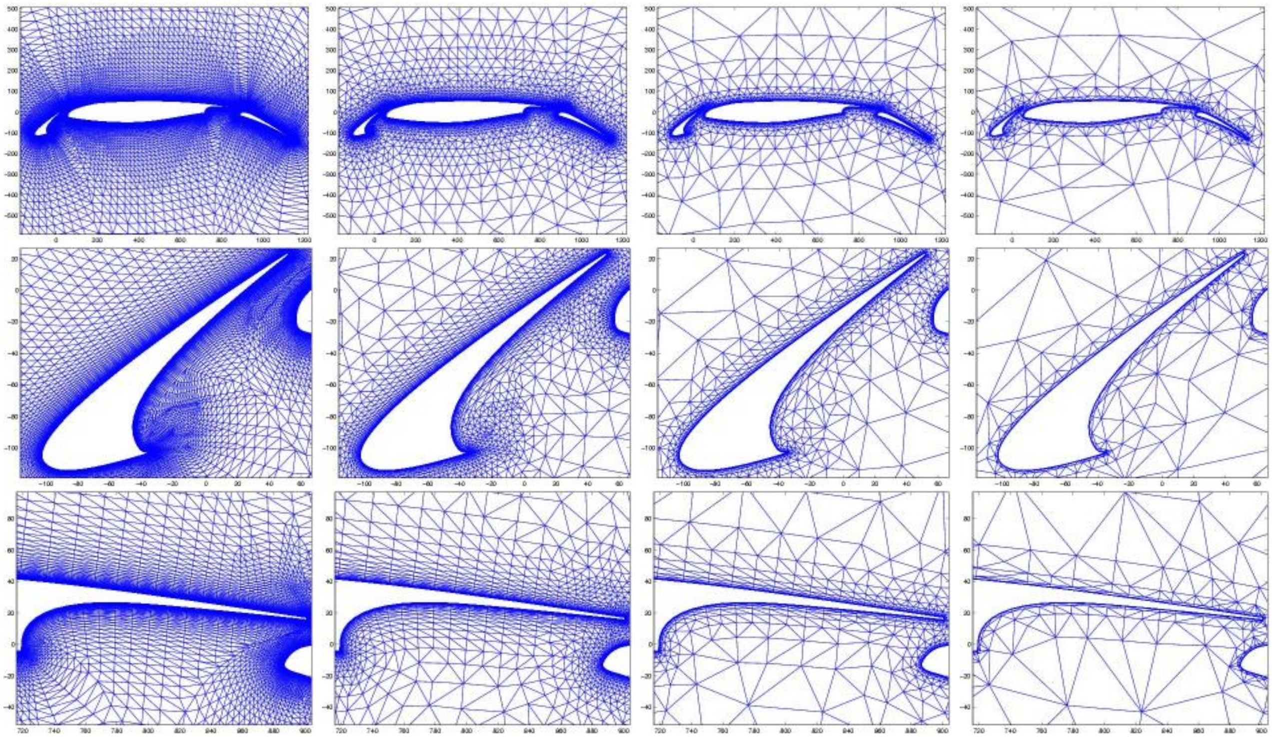

Figure 4: Automatically coarsened highly anisotropic mesh around a multi-body wing

- Starting from a given anisotropic mesh, the first subproblem was how to devise an initial metric field in which the given mesh would appear equilateral. This metric is a maximizer of quality of the original mesh in the space of all possible metric fields.
- The initial metric was then modified in order to coarsen the initial mesh with a factor of at most 2, while tending to compensate for anisotropies, so that the coarser mesh tends to be isotropic.
- Then I modified, in collaboration with Dr. Thierry Coupez (CEMEF), the MTC mesher to be able to work with highly anisotropic metrics and introduce a secondary quality criterion measuring the structuredness of the mesh.

Perspective for future work

Basic operations in mesh generation include measuring the edge-lengths. Rigorous calculations of edge-lengths with respect to a continuous curvilinear metric field involve, however, the calculation of geodesics. This is non-trivial and costly. Instead, most codes suppose that the length of this geodesic is not too different from the length of a linear edge. Unfortunately, for high anisotropies this assumption is no longer true, leading to a breakdown of conventional approach for aspect ratios of more than 10^3. That is why, during my study, I used some cheap fixes to this simplification allowing to pass to higher aspect ratios. In the future work, one should develop a more consistent approach for high anisotropies, either by calculating rapidly the approximations to geodesics, or by using only piecewise smooth metric fields.

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